$$\int_0^{\frac{9}{2}\pi} \frac{\sin x-2}{\cos x+2}\,dx = \int_0^{\pi} \dots dx + \int_{\pi}^{3\pi} + \int_{3\pi}^{\frac{9}{2}\pi} = 2\cdot\int_{-\pi}^{\pi} + \int_0^{\frac{\pi}{2}} \underset{(+),(++)}{\overset{\uparrow}{=}} \log\frac{3}{2} - \frac{26\pi}{3\sqrt{3}}$$

$$\left.\begin{aligned} y=\operatorname{tg}\frac{x}{2} &\Rightarrow \frac{2\,dy}{1+y^2}=dx \\ \frac{1-y^2}{1+y^2}=\cos x;\ &\frac{2y}{1+y^2}=\sin x \end{aligned}\right\}\ x\in(-\pi,\pi)$$

Počítame PF na $(-\pi,\pi)$.

$$\int \frac{\sin x-2}{\cos x+2}\,dx = \int \frac{\frac{2y}{1+y^2}-2}{\frac{1-y^2}{1+y^2}+2}\cdot\frac{2\,dy}{1+y^2} = \int \frac{\frac{-2y^2+2y-2}{1+y^2}}{\frac{y^2+3}{1+y^2}}\cdot\frac{2}{1+y^2}\,dy$$

$$=-4\int \frac{y^2-y+1}{(y^2+1)(y^2+3)}\,dy = -4\int \frac{Ay+B}{y^2+1}+\frac{Cy+D}{y^2+3}\,dy = (*)$$

$$(Ay+B)(y^2+3)+(Cy+D)(y^2+1)=y^2-y+1$$

$$\left.\begin{aligned} y=i:&\ (Ai+B).2=-i \Rightarrow \underline{B=0},\ \underline{A=-\tfrac{1}{2}} \\ y=\sqrt{3}i:&\ (C\sqrt{3}i+D)(-2)=-2-\sqrt{3}i \Rightarrow \underline{D=1},\ \underline{C=\tfrac{1}{2}} \end{aligned}\right\}\ \text{Dosazování kořenů.}$$

$$(*) = \log|y^2+1| - \log|y^2+3| - 4\underbrace{\int\frac{1}{y^2+3}\,dy}_{\frac{1}{3}\int\frac{1}{\left(\frac{y}{\sqrt{3}}\right)^2+1}dy} \overset{C}{=} \log\left|\frac{y^2+1}{y^2+3}\right| - \frac{4}{\sqrt{3}}\operatorname{arctg}\left(\frac{y}{\sqrt{3}}\right)$$

$$\int_{-\pi}^{\pi}\frac{\sin x-2}{\cos x+2}\,dx = \int_{-\infty}^{\infty}\dots dy = \left[\log\left|\frac{y^2+1}{y^2+3}\right| - \frac{4}{\sqrt{3}}\operatorname{arctg}\left(\frac{y}{\sqrt{3}}\right)\right]_{-\infty}^{\infty} = 0-\frac{4}{\sqrt{3}}\frac{\pi}{2} - \left(0+\frac{4}{\sqrt{3}}\frac{\pi}{2}\right) = \cancel{\frac{8}{\sqrt{3}}}-\frac{4}{\sqrt{3}}\pi \quad (++)$$

$$\int_0^{\frac{\pi}{2}}\frac{\sin x-2}{\cos x+2}\,dx = \int_0^1\dots dy = [\dots]_0^1 = \log\frac{1}{2}-\frac{4}{\sqrt{3}}\frac{\pi}{6}-\log\frac{1}{3} = \log\frac{3}{2}-\frac{4}{\sqrt{3}}\cdot\frac{\pi}{6}. \quad (+)$$

Poznámka: Úprava $\int_{-\infty}^{\infty}\frac{y^2-y+1}{(y^2+1)(y^2+3)}\,dy = \int_{-\infty}^{\infty}\frac{2y}{y^2+1}\,dy + \int_{-\infty}^{\infty}\frac{-2y-4}{y^2+3}\,dy$

!! je hrubá chyba. Tyto integrály nejsou konečné $(+\infty+(-\infty))$.